QCD sum rules analysis of the rare radiative $B_c \to D_s^* \gamma$ decay

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Abstract

In this work, the radiative $B_c \to D_s^* \gamma$ decay is investigated in the framework of QCD sum rules. The transition form factors responsible for the decay are calculated. The total branching ratio for this decay is estimated to be in the order of 10^{-5}, so this decay can be measurable at LHC in the near future.

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1 Introduction

The heavy pseudoscalar meson B_c contains two heavy quarks of different flavor. This meson has been discovered in 1998 via the decay mode $B_c \rightarrow J/\psi l^{\pm}\nu$ in 1.8 TeV $p\overline{p}$ collisions, using the CDF detector at the Fermi Lab [1]. The B_c meson constitutes a very rich laboratory for studying various decay channels. There are three classes of decays of B_c meson, namely b-quark decay (when c is spectator), c-quark decay (when b is spectator) and the weak annihilation channels. Because of two heavy quark contents, the B_c decay channels are expected to be very rich in comparison with other B mesons, so investigation of this meson is essential from both theoretical and experimental point of view. The B_c meson decays provide windows for reliable determination of the CKM matrix element V_{cb} and can shed light on new physics beyond the standard model.

At LHC with the luminosity values of $\mathcal{L} = 10^{34}cm^{-2}s^{-1}$ and $\sqrt{s} = 14\text{TeV}$, the number of $B_c^{\pm}$ events is expected to be about $10^8 \sim 10^{10}$ per year, so there are high probability to study not only some rare B_c decays, but also CP violation, T violation and polarization asymmetries. Some possible channels are $B_c \rightarrow l\overline{\nu}\gamma$, $B_c \rightarrow \rho^{+}\gamma$, $B_c \rightarrow K^{*+}\gamma$ and $B_c \rightarrow B_u^{*}l^{+}l^{-}$, $B_c \rightarrow B_u^{*}\gamma$, which have been studied in the frame of light-cone QCD and three point QCD sum rules [2, 3, 4, 13]. A large set of exclusive nonleptonic and semileptonic decays of the B_c meson, which have been studied within a relativistic constituent quark model can be found in [5]. Another possible decay channel of B_c is $B_c^{-} \rightarrow \eta'\ell^{-}\bar{\nu}$ decay, which is studied both for decay rate and lepton polarization asymmetry [6]. We analyzed the radiative $B_c \rightarrow D_s^{*}\gamma$ decay by using QCD sum rules method. Using our calculations at the end, we also analyze the $B_c \rightarrow D^{*}\gamma$ decay by making the necessary changes.

The main quantities in analyzing of $B_c \rightarrow D_s^{*}\gamma$ decay are the form factors.

For the calculation of form factors, relevant to this transition, we need some nonperturbative approaches. Among the nonperturbative approaches, QCD sum rules method has received special attention, because this approach is based on QCD lagrangian. This method has been successfully applied to a wide variety of problems (for a review see [2, 3, 4, 7, 8, 9, 10, 11, 12]).

The $B_c \to D_s^* \gamma$ decay occurs via flavor changing neutral current (FCNC) transition ($b \to s\gamma$) and weak annihilation channels. The b quark decay (electromagnetic penguin) for $B_c \to B_u^* \gamma$ has been calculated in [13] (for more details about the electromagnetic penguin diagram see also [14]). We repeated the similar calculations for our problem and found that the corresponding branching ratio contribution was less important than that of the weak annihilation channel. Note that, the $B_c \to D_s^* \gamma$ decay has been investigated in the perturbative QCD (PQCD) approach [15], relativistic independent quark model (RIQM) [16] and in the framework of pertubative QCD in SM (PQCD), multiscale walking technicolor (MWTCM) and top-color assisted MWTCM (TAMWTCM) models [17]. They found also that the contribution of the weak annihilation was important than that of the electromagnetic penguin in PQCD, RIQM and TAMWTCM. In addition, $B_c \to D^* \gamma$ decay has also been investigated in the relativistic independent quark model (RIQM) [16].

The paper is organized as follows: In section (2) we construct the transition amplitude for the weak annihilation channel in terms of four relevant form factors, where a photon can be radiated from B_c or D_s^*. Two of the relevant form factors for this decay are calculated in [2] in the framework of the light–cone QCD sum rules. In section (3), we calculate the remaining two form factors, when a photon is radiated from D_s^* meson also in light–cone QCD. In section (4), we calculate the transition form factors for electro-
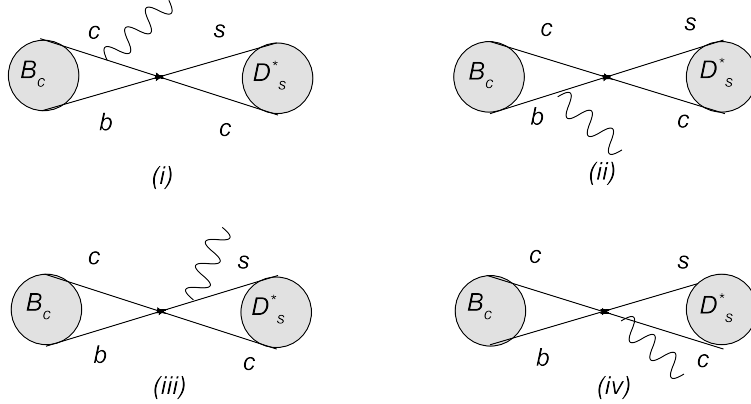
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Figure 1: The weak annihilation mechanism for $B_c \to D_s^* \gamma$

magnetic penguin in the framework of three point QCD sum rules method. Finally, in section (5) numerical analysis, discussion and comparison of our results to those of the other approaches are forwarded and conclusion is presented.

2 Transition amplitude of the weak annihilation for the radiative $B_c \to D_s^* \gamma$ decay

In this section, we concentrated on the main points for obtaining the matrix elements of the radiative decay $B_c \to D_s^* \gamma$ along the lines similar to [18]. The weak annihilation mechanism for this decay is shown in (Fig.1). The transition amplitude for this decay can be written as:

$$M(B_c \to D_s^* \gamma) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* < D_s^*(p)\gamma(q) \mid (\overline{s}\Gamma_\nu c)(\overline{c}\Gamma^\nu b) \mid B_c(p+q) > \quad (1)$$

where $\Gamma_\nu = \gamma_\nu(1 - \gamma_5)$ and p, q and p+q are the momentum of D_s^*, photon and B_c, respectively. Using factorization hypothesis, The matrix element in Eq. (1) can be written in the following form:

$$< D_s^*(p)\gamma(q) \mid (\overline{s}\Gamma_\nu c)(\overline{c}\Gamma^\nu b) \mid B_c(p+q) > \quad = \quad -e\varepsilon^\mu \varepsilon^{(D_s^*)\nu} f_{D_s^*} m_{D_s^*} T_{\mu\nu}^{(B_c)}$$

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$$-ie\varepsilon^{\mu}(p+q)^{\nu}f_{B_c}T_{\mu\nu}^{(D_s^*)} \quad (2)$$

where, the covariant decomposition of hadronic matrix elements $T_{\mu\nu}^{(B_c)}$ and $T_{\mu\nu}^{(D_s^*)}$ are responsible for the emission of photon from initial and final states, $f_{D_s^*}$, f_{B_c} are the leptonic decay constants of D_s^* and B_c mesons, respectively, and ε^{μ} and $\varepsilon^{(D_s^*)\nu}$ are the polarization vectors of a photon and D_s^* meson. The covariant decomposition of hadronic matrix elements are defined by the following two-point correlation functions:

$$T_{\mu\nu}^{(B_c)}(p,q) = i \int d^4x e^{iqx} < 0 \mid T\{j_{\mu}^{em}\overline{c}\Gamma_{\nu}b(0)\} \mid B_c(p+q) > \quad (3)$$

and

$$T_{\mu\nu}^{(D_s^*)}(p,q) = i \int d^4x e^{iqx} < D_s^*(p) \mid T\{j_{\mu}^{em}\overline{s}\Gamma_{\nu}c(0)\} \mid 0 > \quad (4)$$

where j_{μ}^{em} stands for electromagnetic current. Our aim is to construct the $T_{\mu\nu}^{(B_c)}$ and $T_{\mu\nu}^{(D_s^*)}$ in terms of form factors and other physical quantities. Let first focus on $T_{\mu\nu}^{(B_c)}$. This quantity can be written in terms of two independent 4-momenta p and q in general as follows:

$$T_{\mu\nu}^{(B_c)}(p,q) = g_{\mu\nu}a + p_{\mu}q_{\nu}b + q_{\mu}p_{\nu}c + p_{\mu}p_{\nu}d + q_{\mu}q_{\nu}e + \varepsilon_{\nu\mu\lambda\sigma}p^{\lambda}q^{\sigma}F_V^{(B_c)} \quad (5)$$

where a, b, c, d, e and $F_V^{(B_c)}$ are invariant amplitudes. Applying the Ward identity for electromagnetic current to Eq. (5) and using the fact that for a real photon $q^2 = 0$, we rewrite Eq.(5) in the following form:

$$\begin{aligned}
T_{\mu\nu}^{(B_c)}(p,q) &= (g_{\mu\nu}(p.q) - p_{\mu}q_{\nu})iF_A^{(B_c)} + g_{\mu\nu}(p.q)\alpha + p_{\mu}q_{\nu}\beta + q_{\mu}q_{\nu}c \\
&\quad + i\frac{p_{\mu}p_{\nu}}{p.q}f_{B_c} + q_{\mu}q_{\nu}e + \varepsilon_{\nu\mu\lambda\sigma}p^{\lambda}q^{\sigma}F_V^{(B_c)}
\end{aligned} \quad (6)$$

where $F_A^{(B_c)}$, α and β are the new invariant amplitudes. To obtain the relation between α and β, we compare Eqs. (5) and (6), which leads to

$$g_{\mu\nu}a + p_{\mu}q_{\nu}b = (g_{\mu\nu}(p.q) - p_{\mu}q_{\nu})iF_A^{(B_c)} + g_{\mu\nu}(p.q)\alpha + p_{\mu}q_{\nu}\beta \quad (7)$$

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by multiplying both sides of Eq. (7) with q^μ and using $a + (p.q)b = if_{B_c}$ we get the following relation between α and β, which is called the Ward identity.

$$\alpha + \beta = \frac{if_{B_c}}{(p.q)} \tag{8}$$

From Eq. (8), it is clear that α and β can take different choices. Within the scop of the present work, in parallel with[18], we set $\beta = 0$ and $\alpha = \frac{if_{B_c}}{(p.q)}$. Substituting the values of α and β, we obtain

$$
\begin{aligned}
T_{\mu\nu}^{(B_c)}(p,q) &= (g_{\mu\nu}(p.q) - p_\mu q_\nu)iF_A^{(B_c)} + if_{B_c}g_{\mu\nu} + q_\mu q_\nu c \\
&\quad + i\frac{p_\mu p_\nu}{p.q}f_{B_c} + q_\mu q_\nu e + \varepsilon_{\nu\mu\lambda\sigma}p^\lambda q^\sigma F_V^{(B_c)}
\end{aligned} \tag{9}
$$

Using $\varepsilon.q = 0$, $\varepsilon^{(D_s^*)}.p = 0$ and Eq. (9), we obtain the following expression for the first term in Eq. (2) in terms of two form factors $(F_A^{(B_c)}, \ F_V^{(B_c)})$:

$$
\begin{aligned}
e\varepsilon^\mu \varepsilon^{(D_s^*)\nu} f_{D_s^*} m_{D_s^*} T_{\mu\nu}^{(B_c)} &= ef_{D_s^*} m_{D_s^*}\{[(\varepsilon.\varepsilon^{(D_s^*)})(p.q) - (\varepsilon.p)(\varepsilon^{(D_s^*)}.q)]iF_A^{(B_c)} \\
&\quad + if_{B_c}(\varepsilon.\varepsilon^{(D_s^*)}) + \varepsilon_{\nu\mu\lambda\sigma}\varepsilon^{(D_s^*)\nu}\varepsilon^\mu p^\lambda q^\sigma F_V^{(B_c)}\} \tag{10}
\end{aligned}
$$

Omitting the details for the calculation $T_{\mu\nu}^{(D_s^*)}$, we get the following result for the second term in the Eq. (2):

$$
\begin{aligned}
ie\varepsilon^\mu(p+q)^\nu f_{B_c} T_{\mu\nu}^{(D_s^*)} &= ief_{B_c}\{[(\varepsilon.\varepsilon^{(D_s^*)})(p.q) - (\varepsilon.p)(\varepsilon^{(D_s^*)}.q)]iF_A^{(D_s^*)} \\
&\quad + f_{D_s^*} m_{D_s^*}(\varepsilon.\varepsilon^{(D_s^*)}) + \varepsilon_{\nu\mu\lambda\sigma}\varepsilon^{(D_s^*)\nu}\varepsilon^\mu p^\lambda q^\sigma F_V^{(D_s^*)}\} \tag{11}
\end{aligned}
$$

where, $F_A^{(D_s^*)}$ and $F_V^{(D_s^*)}$ are two form factors of D_s^*. Now, we can write the transition amplitude for the radiative $B_c \to D_s^*\gamma$ decay in terms of four form factors $F_A^{(B_c)}$, $F_V^{(B_c)}$, $F_A^{(D_s^*)}$ and $F_V^{(D_s^*)}$, as follows:

$$
\begin{aligned}
M(B_c \to D_s^*\gamma) &= e\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^*[-f_{D_s^*} m_{D_s^*}\{[(\varepsilon.\varepsilon^{(D_s^*)})(p.q) - (\varepsilon.p)(\varepsilon^{(D_s^*)}.q)]iF_A^{(B_c)} \\
&\quad + if_{B_c}(\varepsilon.\varepsilon^{(D_s^*)}) + \varepsilon_{\nu\mu\lambda\sigma}\varepsilon^{(D_s^*)\nu}\varepsilon^\mu p^\lambda q^\sigma F_V^{(B_c)}\} \\
&\quad - if_{B_c}\{[(\varepsilon.\varepsilon^{(D_s^*)})(p.q) - (\varepsilon.p)(\varepsilon^{(D_s^*)}.q)]iF_A^{(D_s^*)} \\
&\quad + f_{D_s^*} m_{D_s^*}(\varepsilon.\varepsilon^{(D_s^*)}) + \varepsilon_{\nu\mu\lambda\sigma}\varepsilon^{(D_s^*)\nu}\varepsilon^\mu p^\lambda q^\sigma F_V^{(D_s^*)}\}] \tag{12}
\end{aligned}
$$

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The form factors $F_A^{(B_c)}$ and $F_V^{(B_c)}$ corresponding to the emission of the photon from b and c quark (see Fig. 1 i, ii), are calculated in [2]. Therefore, we will concentrate on the calculation of the form factors $F_A^{(D_s^*)}$ and $F_V^{(D_s^*)}$ (see Fig.1 iii, iv).

3   Light cone QCD sum rules for the form factors $F_A^{(D_s^*)}$ and $F_V^{(D_s^*)}$

Based on the general idea on QCD sum rules method, we will calculate the transition form factors by equating the representation of a suitable correlator in hadronic and quark–gluon languages. For this aim, we consider the following correlation function:

$$\Pi_{\mu\nu}(p,q) = i \int d^4x e^{iQx} < \gamma(q) \mid T\{\overline{c}(x)\gamma_\mu(1 - \gamma_5)s(x)\overline{s}(0)\gamma_\nu c(0)\} \mid 0 > \tag{13}$$

Here, q and p are the momentum values of photon and D_s^*, respectively and $Q = p + q$ is the transferred momentum. Now, we insert the hadronic state $D_s^*(p)$ to Eq. (13). This can be re-written as:

$$\Pi_{\mu\nu}(p,q) = \frac{< \gamma(q) \mid \overline{c}\gamma_\mu(1 - \gamma_5)s \mid D_s^*(p) >< D_s^*(p) \mid \overline{s}\gamma_\nu c\} \mid 0 >}{m_{D_s^*}^2 - p^2}. \tag{14}$$

The second term in Eq. (14), by definition is:

$$< D_s^*(p) \mid \overline{s}\gamma_\nu c \mid 0 >= f_{D_s^*} m_{D_s^*} \varepsilon_\nu^{(D_s^*)} \tag{15}$$

From Fig. 1 (iii, iv) and due to the fact that parity, Lorentz and gauge invariance are musts. We can write the matrix element for the emission of the photon from D_s^* meson as:

$$
\begin{aligned}
< \gamma(q) \mid \overline{c}\gamma_\mu(1 - \gamma_5)s \mid D_s^*(p) > \;=\; & e\{i\varepsilon_{\mu\alpha\beta\sigma}\varepsilon^\alpha\varepsilon^{(D_s^*)\beta}q^\sigma \frac{F_V^{(D_s^*)}(Q^2)}{m_{D_s^*}^2} \\
& + \; [\varepsilon_\mu(\varepsilon^{(D_s^*)}.q) - (\varepsilon.\varepsilon^{(D_s^*)})q_\mu]\frac{F_A^{(D_s^*)}(Q^2)}{m_{D_s^*}^2}\}
\end{aligned} \tag{16}
$$

Substituting Eqs. (15) and (16) to (14), we have

$$
\begin{aligned}
\Pi_{\mu\nu}(p,q) &= \frac{ef_{D_s^*}m_{D_s^*}}{m_{D_s^*}^2 - p^2}\{i\varepsilon_{\mu\alpha\beta\sigma}\varepsilon^\alpha\varepsilon_\nu^{(D_s^*)}\varepsilon^{(D_s^*)\beta}q^\sigma\frac{F_V^{(D_s^*)}(Q^2)}{m_{D_s^*}^2} \\
&+ [\varepsilon_\mu\varepsilon_\nu^{(D_s^*)}(\varepsilon^{(D_s^*)}.q) - (\varepsilon.\varepsilon^{(D_s^*)})\varepsilon_\nu^{(D_s^*)}q_\mu]\frac{F_A^{(D_s^*)}(Q^2)}{m_{D_s^*}^2}\}
\end{aligned}
\tag{17}
$$

Summation over polarization of D_s^* meson is performed by using:

$$
\varepsilon_\nu^{(D_s^*)}\varepsilon_\beta^{(D_s^*)} = -g_{\nu\beta} + \frac{p_\nu p_\beta}{m_{D_s^*}^2}
\tag{18}
$$

After performing the standard calculations for the phenomenological part, we get:

$$
\Pi_{\mu\nu}(p,q) = \frac{ef_{D_s^*}m_{D_s^*}}{m_{D_s^*}^2 - p^2}\{i\varepsilon_{\mu\nu\alpha\sigma}\varepsilon^\alpha q^\sigma\frac{F_V^{(D_s^*)}(Q^2)}{m_{D_s^*}^2} + [q_\mu\varepsilon_\nu - \varepsilon_\mu q_\nu]\frac{F_A^{(D_s^*)}(Q^2)}{m_{D_s^*}^2}\}
\tag{19}
$$

The theoretical part (QCD side) of the correlator is calculated by means of OPE up to operators having dimension $d = 5$ in deep Euclidean space, where both p^2 and Q^2 are large and negative. It is determined by the bare–loop (fig. 2(a, b)) and the power corrections from the operators with $d = 3$, $<\overline{\psi}\psi>$, $d = 4$, $m_s <\overline{\psi}\psi>$, $d = 5$, $m_0^2 <\overline{\psi}\psi>$ (fig. 2(c, d, e)) and the photon interaction with a soft quark line (fig. 2f). In calculating the bare-loop and nonperturbative correction contributions, we first write the Lorentz decomposition of the correlator as:

$$
\Pi_{\mu\nu}(p,q) = i\varepsilon_{\mu\nu\alpha\sigma}\varepsilon^\alpha q^\sigma\Pi_1 + [q_\mu\varepsilon_\nu - \varepsilon_\mu q_\nu]\Pi_2
\tag{20}
$$

and the dispersion representation (Cutkosky method) for the coefficients of corresponding Lorentz structures appearing in the $\Pi_{\mu\nu}(p,q)$ as follows:

$$
\Pi_{1,2}(p,q) = \int ds\frac{\rho_{1,2}(s,p^2)}{s - Q^2} + \text{subtraction terms}
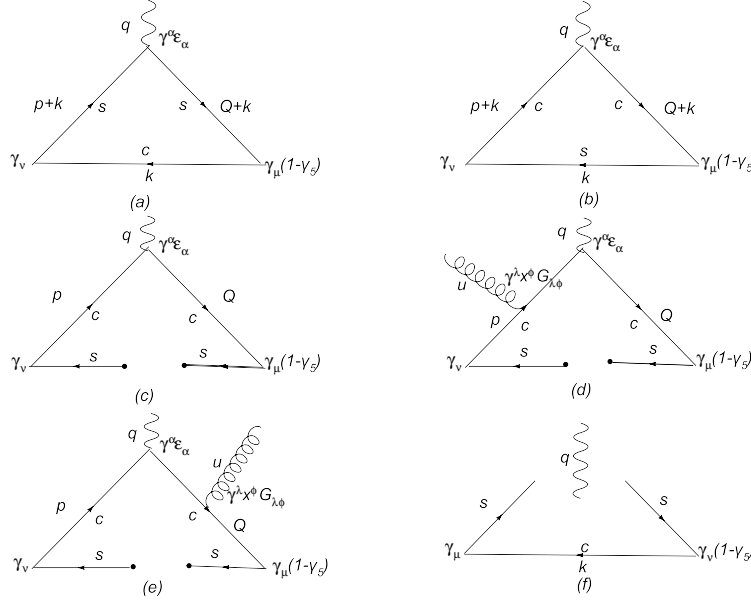\tag{21}
$$

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Figure 2: Feynmen diagrams for bare-loop (a, b), power corrections from the operators with 3, 4 and 5 dimensions (c, d, e) and propagation of the soft quark in electromagnetic field (f)

where $\rho_{1,2}(s, p^2)$ are spectral density corresponding to two structures in $\Pi_{\mu\nu}(p, q)$ and subtraction terms stand for corrections. To calculate $\rho_{1,2}(s, p^2)$, we consider Feynmen diagrams in Fig. 2(a, b). For instance, for the contribution of diagram (a) we get

$$
\begin{aligned}
T_{\mu\nu} = & \ eN_cQ_s \int \frac{d^4k}{(2\pi)^4} \\
& \{Tr[\frac{i(\not{k} + m_c)}{k^2 - m_c^2}\gamma_\mu(1 - \gamma_5)\frac{i(\not{Q}+ \not{k} + m_s)}{(Q + k)^2 - m_s^2} \not{\varepsilon}\frac{i(\not{p}+ \not{k} + m_s)}{(p + k)^2 - m_s^2}\gamma_\nu]\}
\end{aligned}
$$

(22)

With the help of the above equations, we obtain the following expressions corresponding to the coefficients of the structures $i\varepsilon_{\mu\nu\alpha\sigma}\varepsilon^\alpha q^\sigma$ and $[q_\mu\varepsilon_\nu - \varepsilon_\mu q_\nu]$:

$$
\begin{aligned}
T_1 = & \ \frac{-eN_cQ_s}{4\pi^2}\{\int_0^1 dx x \int_0^1 dy[m_c m_s - m_s^2 x\overline{y} + p^2(X_1\overline{y} + X_3) - (p.q)(X_2\overline{y}^2 \\
& - 4x^2\overline{x}y + 2x^2\overline{x}y^2 - 4x\overline{x}y)]\int_0^\infty d\alpha e^{-\alpha\Delta}\},
\end{aligned}
$$

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$$
\begin{aligned}
T_2 &= \frac{eN_cQ_s}{16\pi^2}\{\int_0^1 dx\,x \int_0^1 dy[m_cm_s - m_s^2 x\overline{y} + p^2(X_1\overline{y} - X_3) - (p.q)X_3(2\overline{y} \\
&+ 4\overline{x}y)]\int_0^\infty d\alpha e^{-\alpha\Delta}\}
\end{aligned}
\tag{23}
$$

where $\overline{x}(\overline{y}) = 1 - x(y)$, $\Delta = -p^2 x\overline{x}y - Q^2 x\overline{x}\,\overline{y} + m_c^2\overline{x} + m_s^2 x$, $X_1 = x - 2x^2 + x^3$, $X_2 = x^2 - 2x^3$, $X_3 = -x + x^2$

In this calculation, we have also used the exponential representation for the denominator as:

$$
\frac{1}{\Delta^n} = \frac{1}{(n-1)!}\int_0^\infty d\alpha\,\alpha^{n-1}e^{-\alpha\Delta}
\tag{24}
$$

Next, we apply the double Borel operator $\hat{B}(M_1^2)\hat{B}(M_2^2)$ on $T_{(1,2)}$ and we get:

$$
\begin{aligned}
\tilde{T}_1 &= \frac{-eN_cQ_s}{4\pi^2}\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}\int_0^1 dx\frac{1}{\overline{x}}e^{\frac{1}{x\overline{x}}(m_c^2\overline{x}+m_s^2 x)(\sigma_1+\sigma_2)}[m_cm_s - m_s^2 x\frac{\sigma_2}{\sigma_1+\sigma_2} \\
&+ p^2((x^3-2x^2)\frac{\sigma_2}{\sigma_1+\sigma_2} + x^2 - x\frac{\sigma_1}{\sigma_1+\sigma_2}) - (p.q)(x^2 - 2x^3 + (3x^2 \\
&- 4x^3)\frac{\sigma_1^2}{(\sigma_1+\sigma_2)^2} + (8x^3 - 2x^2 - 4x)\frac{\sigma_1}{\sigma_1+\sigma_2})], \\
\tilde{T}_2 &= \frac{eN_cQ_s}{16\pi^2}\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}\int_0^1 dx\frac{1}{\overline{x}}e^{\frac{1}{x\overline{x}}(m_c^2\overline{x}+m_s^2 x)(\sigma_1+\sigma_2)}[m_cm_s - m_s^2 x\frac{\sigma_2}{\sigma_1+\sigma_2} \\
&+ p^2((x + x^3 - 2x^2)\frac{\sigma_2}{\sigma_1+\sigma_2} - x^2 + x) - (p.q)(-2x + 2x^2)\frac{\sigma_2}{\sigma_1+\sigma_2} \\
&- 2x^3\frac{\sigma_2^2}{(\sigma_1+\sigma_2)^2} + (8x^2 - 4x^3 - 4x)\frac{\sigma_1}{\sigma_1+\sigma_2}) + (2x^3 - 2x^2)\frac{\sigma_1^2}{(\sigma_1+\sigma_2)^2}]
\end{aligned}
\tag{25}
$$

where $\sigma_1 = \frac{1}{M_1^2}$ and $\sigma_2 = \frac{1}{M_2^2}$.

In deriving Eq. (25), we use the definition

$$
\hat{B}(M^2)e^{-\alpha p^2} = \delta(1 - \alpha M^2)
\tag{26}
$$

For the determination of the spectral density, we apply the Borel transformations to $\tilde{T}_1$ and $\tilde{T}_2$ [19] and we obtain:

$$
\varrho_{1,2}(s,t,p^2) = \frac{1}{st}\hat{B}(\frac{1}{s},\sigma_1)\hat{B}(\frac{1}{t},\sigma_2)\frac{\tilde{T}_{1,2}}{\sigma_1\sigma_2}
\tag{27}
$$

In this step, we use the following relations:

$$\sigma^n e^{-\alpha\sigma} = (-\frac{d}{d\alpha})^n e^{-\alpha\sigma}, \tag{28}$$

$$\rho_{1,2}(s,p^2) = \int dt \frac{\varrho_{1,2}(s,t,p^2)}{t-p^2}, \tag{29}$$

and

$$\hat{B}(\frac{1}{s},\sigma_1)\hat{B}(\frac{1}{t},\sigma_2)e^{-\alpha(\sigma_1+\sigma_2)} = \delta(1-\frac{\alpha}{s})\delta(1-\frac{\alpha}{t}) \tag{30}$$

Then, we get the following expressions for the two spectral densities, as follows:

$$
\begin{aligned}
\rho_1(s,p^2) &= \frac{eN_cQ_s}{4\pi^2}\frac{1}{(s-p^2)^3}\int_{x_0}^{x_1}dx\frac{1}{\overline{x}}\{(m_cm_s-m_s^2x)(s-p^2)^2 + p^2[(x^3-x^2) \\
&\quad (s-p^2)^2 - x(p^2+\frac{m_c^2}{x}+\frac{m_s^2}{\overline{x}})(s-p^2)] - \frac{1}{2}(m_s^2-p^2)[(x^2-2x^3) \\
&\quad (s-p^2)^2 - 2(3x^2-4x^3)(p^2+\frac{m_c^2}{x}+\frac{m_s^2}{\overline{x}})^2 + (8x^3-2x^2-4x)(p^2 \\
&\quad +\frac{m_c^2}{x}+\frac{m_s^2}{\overline{x}})(s-p^2)]\}, \\
\rho_2(s,p^2) &= \frac{eN_cQ_s}{16\pi^2}\frac{1}{(s-p^2)^3}\int_{x_0}^{x_1}dx\frac{1}{\overline{x}}\{[m_cm_s-m_s^2x+p^2(2x-3x^2+x^3)] \\
&\quad (s-p^2)^2 - \frac{1}{2}(m_s^2-p^2)[(-2x+2x^2-4x^3)(s-p^2)^2 + (8x^2-4x^3 \\
&\quad -4x)(p^2+\frac{m_c^2}{x}+\frac{m_s^2}{\overline{x}})(s-p^2) - 2(2x^3-2x^2)(p^2+\frac{m_c^2}{x}+\frac{m_s^2}{\overline{x}})^2]\}
\end{aligned}
\tag{31}
$$

where the integration region is determined by the following inequality:

$$sx\overline{x} - (m_c^2\overline{x}+m_s^2x) \geq 0 \tag{32}$$

Similar to above calculations for diagram (a), we have repeated the entire calculations for diagram (b). Finally, we get the following results for the

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spectral densities:

$$\rho_1(s,p^2) = \frac{eN_c}{48\pi^2}\frac{1}{(s-p^2)^3}\{Q_s[\lambda\{3(-3+5\alpha-5\beta)p^6 + (28+\alpha(-103$$

$$+\alpha(17+4\alpha))+52\beta-\alpha(61+8\alpha)\beta+4(10+\alpha)\beta^2)p^4 s - 3(-2$$

$$+16\alpha^3+\beta-31\beta^2-\alpha^2(75+32\beta)+\alpha(39+2\beta(41+8\beta))+7\lambda^2$$

$$)p^2 s^2 + \alpha(1-86\alpha^2+\beta-86\beta^2+\alpha(67+172\beta)+18\lambda^2)s^3\}+6\{$$

$$p^6+2m_c m_s(s-p^2)^2+(2-11\alpha+4\beta)p^4 s(-1+26\alpha^2+2\beta^2$$

$$-2\alpha(7+11\beta))p^2 s^2-\alpha(1+18\alpha^2+2\beta(1+\beta)-4\alpha(4+5\beta))s^3\}$$

$$ln\frac{1+\alpha-\beta-\lambda}{1+\alpha-\beta+\lambda}]+Q_c[\lambda\{3(-3+5\beta-5\alpha)p^6+(28+\beta(-103$$

$$+\beta(17+4\beta))+52\alpha-\beta(61+8\beta)\alpha+4(10+\beta)\alpha^2)p^4 s - 3(-2$$

$$+16\beta^3+\alpha-31\alpha^2-\beta^2(75+32\alpha)+\beta(39+2\alpha(41+8\alpha))+7\lambda^2$$

$$)p^2 s^2 + \beta(1-86\beta^2+\alpha-86\alpha^2+\beta(67+172\alpha)+18\lambda^2)s^3\}+6\{$$

$$p^6+2m_c m_s(s-p^2)^2+(2-11\beta+4\alpha)p^4 s(-1+26\beta^2+2\alpha^2$$

$$-2\beta(7+11\alpha))p^2 s^2-\beta(1+18\beta^2+2\alpha(1+\alpha)-4\beta(4+5\alpha))s^3\}$$

$$ln\frac{1+\beta-\tilde{\alpha}-\lambda}{1+\beta-\alpha+\lambda}]\},$$

$$\rho_2(s,p^2) = \frac{eN_c}{96\pi^2}\frac{1}{(s-p^2)^3}\{Q_s[\lambda\{-(29+2\alpha^2+\beta(17+2\beta)-\alpha(19+4\beta))p^6$$

$$+(4\alpha^3+\alpha(5+\beta)(5+4\beta)-\alpha^2(23+8\beta)+6(8+\beta-\beta^2))p^4 s$$

$$+(-23+2\alpha^3-5\beta(1+4\beta)+\alpha(-9+2\beta(21+\beta))+\alpha^2(-30$$

$$-4\beta+\frac{48}{\lambda^2-(1+\alpha-\beta)^2}))p^2 s^2+\alpha(13+\alpha(-5+22\alpha)+\beta$$

$$-44\alpha\beta+22\beta^2)s^3\}+6\{-2p^6+m_c m_s(s-p^2)^2+(4+5\alpha)p^4 s$$

$$-2(1+\alpha+4\alpha^2-2\alpha\beta)p^2 s^2+\alpha(1+4\alpha(\alpha-\beta))s^3\}$$

$$ln\frac{1-\alpha+\beta-\lambda}{1-\alpha+\beta+\lambda}]+Q_c[\lambda\{-(29+2\beta^2+\alpha(17+2\alpha)-\beta(19+4\alpha))$$

$$p^6+(4\beta^3+\beta(5+\alpha)(5+4\alpha)-\beta^2(23+8\alpha)+6(8+\alpha-\alpha^2))p^4 s$$

$$+(-23+2\beta^3-5\alpha(1+4\alpha)+\beta(-9+2\alpha(21+\alpha))+\beta^2(-30$$

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$$-4\alpha + \frac{48}{\lambda^2 - (1 + \beta - \alpha)^2}))p^2s^2 + \beta(13 + \beta(-5 + 22\beta) + \alpha$$
$$-44\beta\alpha + 22\alpha^2)s^3\} + 6\{-2p^6 + m_c m_s(s - p^2)^2 + (4 + 5\beta)p^4s$$
$$-2(1 + \beta + 4\beta^2 - 2\beta\alpha)p^2s^2 + \beta(1 + 4\beta(\beta - \alpha))s^3\}$$
$$ln\frac{1 - \beta + \alpha - \lambda}{1 - \beta + \alpha + \lambda}]\} \tag{33}$$

where $\alpha = \frac{m_s^2}{s}$ and $\lambda = \sqrt{1 + \alpha^2 + \beta^2 - 2\alpha - 2\beta - 2\alpha\beta}$.

The next step is to calculate contributions coming from the power correction terms. After standard but lengthy calculations for the contributions of the diagrams (c, d, e), we get:

$$
\begin{aligned}
\Pi_1(p, q)^{(c,d,e)} &= \frac{m_c}{r'r} < \overline{s}s > -\frac{m_s}{2} < \overline{s}s > [\frac{m_c^2}{r'^2r} + \frac{1}{r'r} + \frac{m_c^2}{r'r^2}] + \frac{m_s^2}{2} < \overline{s}s > [\frac{2m_c^3}{r'^3r} \\
&\quad + \frac{2m_c^3}{r'^2r^2} + \frac{m_c}{r'^2r} + \frac{m_c}{r'r^2} + \frac{2m_c^3}{r'r^3}] - \frac{m_0^2}{4} < \overline{s}s > [\frac{2m_c^3}{r'^3r} + \frac{2m_c^3}{r'^2r^2} \\
&\quad + \frac{2m_c}{3r'^2r} + \frac{m_c}{r'r^2} + \frac{2m_c^3}{r'r^3}], \\
\Pi_2(p, q)^{(c,d,e)} &= \frac{-m_c}{r'r} < \overline{s}s > +\frac{m_s}{2} < \overline{s}s > [\frac{m_c^2}{r'^2r} + \frac{m_c^2}{r'r^2}] - \frac{m_s^2}{2} < \overline{s}s > [\frac{2m_c^3}{r'^3r} \\
&\quad + \frac{2m_c^3}{r'^2r^2} + \frac{m_c}{r'^2r} + \frac{m_c}{r'r^2} + \frac{2m_c^3}{r'r^3}] + \frac{m_0^2}{4} < \overline{s}s > [\frac{2m_c^3}{r'^3r} + \frac{2m_c^3}{r'^2r^2} \\
&\quad + \frac{2m_c}{3r'^2r} + \frac{m_c}{r'r^2} + \frac{2m_c^3}{r'r^3}] \tag{34}
\end{aligned}
$$

where $r^2 = p^2 - m_c^2$ and $r'^2 = Q^2 - m_c^2$. Finally, we calculate the contribution of diagram (f). For the calculation of this diagram corresponding to the propagation of the soft quark in the external electromagnetic field, we use the light-cone expansion for the non–local operators. After contracting the c quark lines in

$$\Pi_{\mu\nu}(p, q) = i \int d^4x e^{iQx} < \gamma(q) \mid T\{\overline{s}(0)\gamma_\mu c(0)\overline{c}(x)\gamma_\nu(1 - \gamma_5)s(x)\} \mid 0 > \tag{35}$$

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we obtain

$$\Pi_{\mu\nu}(p,q) = i \int d^4x \frac{d^4k}{(2\pi)^2} \frac{e^{i(Q-k)x}}{m_c^2 - k^2} < \gamma(q) \mid \overline{s}\gamma_\mu(\not{k} + m_c)\gamma_\nu(1 - \gamma_5)s\} \mid 0 > \tag{36}$$

To calculate the matrix element appearing in the above equation, we use the following identities:

$$\begin{aligned}
\gamma_\mu\gamma_\nu &= g_{\mu\nu} + i\sigma_{\mu\nu}, \\
\gamma_\mu\gamma_\nu\gamma_5 &= g_{\mu\nu}\gamma_5 - \frac{i}{2}\varepsilon_{\mu\nu\alpha\sigma}\sigma_{\alpha\sigma}, \\
\gamma_\mu\gamma_\alpha\gamma_\nu &= g_{\mu\alpha}\gamma_\nu + g_{\alpha\nu}\gamma_\nu - g_{\mu\nu}\gamma_\alpha + i\varepsilon_{\mu\nu\alpha\sigma}\gamma_\sigma\gamma_5
\end{aligned} \tag{37}$$

and photon distribution amplitudes (DA's) for twist 2, 3 and 4 [20, 21]:

$$\begin{aligned}
< \gamma(q) \mid \overline{s}\gamma_\nu s \mid 0 > &= -\frac{Q_s}{2} f_{3\gamma} \int_0^1 du \overline{\psi}^{(V)}(u) x^\theta F_{\theta\nu}(ux) \\
< \gamma(q) \mid \overline{s}\gamma_\alpha\gamma_5 s \mid 0 > &= -\frac{iQ_s}{4} f_{3\gamma} \int_0^1 du \psi^{(A)}(u) x^\theta \tilde{F}_{\theta\alpha}(ux) \\
< \gamma(q) \mid \overline{s}\sigma_{\alpha\beta} s \mid 0 > &= Q_s < \overline{s}s > \int_0^1 du \phi(u) F_{\alpha\beta}(ux) \\
&\quad + \frac{Q_s < \overline{s}s >}{16} \int_0^1 du x^2 \mathcal{A}(u) F_{\alpha\beta}(ux) \\
&\quad + \frac{Q_s < \overline{s}s >}{8} \int_0^1 du \mathcal{B}(u) x^\rho (x_\beta F_{\alpha\rho}(ux) - x_\alpha F_{\beta\rho}(ux))
\end{aligned} \tag{38}$$

where $F_{\mu\nu}$ is the field strength tensor of the electromagnetic field, which is defined by

$$\begin{aligned}
F_{\mu\nu}(x) &= i(\varepsilon_\nu q_\mu - \varepsilon_\mu q_\nu)e^{iqx} \\
\tilde{F}_{\mu\nu}(x) &= \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}(x)
\end{aligned} \tag{39}$$

The asymptotic expression for the photon wave function $\phi(u)$ in terms of magnetic susceptibility of the quark condensate, $\chi(\mu)$, at a re–normalization

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scale ($\mu = 1 \ GeV^2$) is defined by:

$$\phi(u) = \chi(\mu)u(1-u) \qquad (40)$$

Other functions used in Eq. (38) are defined by [20, 21]

$$
\begin{aligned}
\overline{\psi}^{(V)}(u) &= -20u(1-u)(2u-1) + \frac{15}{16}(\omega_\gamma^A - 3\omega_\gamma^V)u(1-u)(2u-1) \\
&\quad \times (7(2u-1)^2 - 3), \\
\psi^{(A)}(u) &= (1 - (2u-1)^2)(5(2u-1)^2 - 1) \times \frac{5}{2}(1 + \frac{9}{16}\omega_\gamma^V - \frac{3}{16}\omega_\gamma^A), \\
\mathcal{A}(u) &= 40u(1-u)(3k - k^+ + 1) + 8(\xi_2^+ - 3\xi_2)[u(1-u)(2 + 13u(1-u)) \\
&\quad + 2u^3(10 - 15u + 6u^2)lnu + 2(1-u)^3(10 - 15(1-u) + 6(1 \\
&\quad - u^2))ln(1-u)], \\
\mathcal{B}(u) &= 40\int_0^u d\alpha(4-\alpha)(1 + 3k^+)[\frac{-1}{2} + \frac{3}{2}(2\alpha - 1)^2] \qquad (41)
\end{aligned}
$$

where k, k^+, ξ_2, ξ_2^+ and $f_{3\gamma}$ are constants (see [20, 21]). Using the above relations in Eq. (36), we obtain:

$$
\begin{aligned}
\Pi_{\mu\nu}(p,q) &= -\int d^4x \frac{d^4k}{(2\pi)^2} \frac{e^{i(p-k)x}}{m_c^2 - k^2} \Big\{ \frac{-Q_s}{2} f_{3\gamma} k_\mu \int_0^1 du\overline{\psi}^{(V)}(u)x^\sigma F_{\sigma\nu} - \frac{Q_s}{2} f_{3\gamma} k_\nu \\
&\quad \times \int_0^1 du\overline{\psi}^{(V)}(u)x^\sigma F_{\sigma\mu} - \frac{Q_s}{2} f_{3\gamma} g_{\mu\nu} k_\alpha \int_0^1 du\overline{\psi}^{(V)}(u)x^\sigma F_{\sigma\alpha} + \frac{Q_s}{4} f_{3\gamma} \\
&\quad \times k_\alpha \int_0^1 du\psi^{(A)}(u)[-x^\alpha F_{\mu\nu} + x_\mu F_{\alpha\nu} + x_\nu F_{\mu\alpha}] + \frac{iQ_s}{8} f_{3\gamma}[k_\mu \varepsilon_{\theta\nu\eta\lambda} \\
&\quad + k_\nu \varepsilon_{\theta\mu\eta\lambda} - g_{\mu\nu} k_\alpha \varepsilon_{\theta\alpha\eta\lambda}] \int_0^1 du\psi^{(A)}(u)x^\theta F_{\eta\lambda} + \frac{iQ_s}{2} f_{3\gamma} k_\alpha \varepsilon_{\mu\nu\alpha\sigma} \\
&\quad \int_0^1 du\overline{\psi}^{(V)}(u)x^\theta F_{\theta\sigma} + im_c[Q_s < \overline{s}s > \int_0^1 du\phi(u)F_{\mu\nu}(ux) \\
&\quad + \frac{Q_s < \overline{s}s >}{16} \int_0^1 dux^2\mathcal{A}(u)F_{\mu\nu}(ux) + \frac{Q_s < \overline{s}s >}{8} \int_0^1 du\mathcal{B}(u)x^\sigma \\
&\quad \times (x_\nu F_{\mu\sigma}(ux) - x_\mu F_{\nu\sigma}(ux))] + \frac{m_c}{2}Q_s < \overline{s}s > \varepsilon_{\mu\nu\alpha\sigma}[\int_0^1 du\phi(u) \\
&\quad \times F_{\alpha\sigma}(ux) + \frac{1}{16} \int_0^1 dux^2\mathcal{A}(u)F_{\alpha\sigma}(ux) + \frac{1}{8} \int_0^1 du\mathcal{B}(u)x^\theta(x_\sigma F_{\alpha\theta}(ux) \\
&\quad - x_\alpha F_{\sigma\theta}(ux))]\Big\} \qquad (42)
\end{aligned}
$$

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After performing integration over x and k, the following results corresponding to the coefficients of two invariant structures $i\varepsilon_{\mu\nu\alpha\sigma}\varepsilon^{\alpha}q^{\sigma}$ and $[q_{\mu}\varepsilon_{\nu} - \varepsilon_{\mu}q_{\nu}]$ are obtained as follows:

$$
\begin{aligned}
\Pi_1(p,q)^{(f)} &= \frac{m_c Q_s < \bar{s}s >}{2} [\int_0^1 du\phi(u)\frac{1}{p^2 - m_c^2} - \frac{1}{16}\int_0^1 du\mathcal{A}(u)(\frac{10}{(p^2 - m_c^2)^2} \\
&\quad + \frac{8m_c^2}{(p^2 - m_c^2)^3})], \\
\Pi_2(p,q)^{(f)} &= \frac{-Q_s}{4}f_{3\gamma}\int_0^1 du\psi^{(A)}(u)(\frac{1}{p^2 - m_c^2} + \frac{2m_c^2}{(p^2 - m_c^2)^2}) - m_c Q_s < \bar{s}s > \\
&\quad [\int_0^1 du\phi(u)\frac{1}{p^2 - m_c^2} - \frac{1}{16}\int_0^1 du\mathcal{A}(u)(\frac{10}{(p^2 - m_c^2)^2} + \frac{8m_c^2}{(p^2 - m_c^2)^3})]
\end{aligned}
$$

$$(43)$$

These results are the final results of the QCD part (OPE expression) of the correlator. The next step is to equate Eq. (20) and Eq. (19) (the physical or phenomenological side of the correlation function) and perform the Borel transformation, with respect to the momentum of D_s^* meson ($p^2 \to M_B^2$), in order to suppress the contributions of higher states and continuum. We obtain the following sum rules for the transition form factors, namely:

$$
F_{V,A}^{(D_s^*)}(Q^2) = \frac{m_{D_s^*}}{f_{D_s^*}}e^{\frac{m_{D_s^*}^2}{M_B^2}}\hat{B}\{\int_{(m_c+m_s)^2}^{s_0}ds\frac{\rho_{1,2}(s,p^2)}{s - Q^2} + \Pi_{1,2}^{c+d+e+f}\} \tag{44}
$$

where V and A are correspond to 1 and 2 in r. h. s., respectively. In Eq. (44), in order to subtract the contributions of the higher states and the continuum, quark-hadron duality assumption is used, i.e. it is assumed that

$$
\rho^{higher\ states}(s) = \rho^{OPE}(s)\theta(s - s_0) \tag{45}
$$

In the calculations, the following rule for the Borel transformation is used:

$$
\hat{B}\frac{1}{(p^2 - s)^n} = (-1)^n\frac{e^{\frac{-s}{M_B^2}}}{\Gamma(n)(M_B^2)^{n-1}} \tag{46}
$$

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4 QCD sum rules for the form factors induced by electromagnetic penguin

The effective Hamiltonian for the $b \to s\gamma$ transition can be written as follows:

$$H = -\frac{G_F e}{4\pi^2\sqrt{2}}V_{tb}V_{ts}^*C_7(\mu)\overline{s}\sigma_{\mu\nu}[m_b\frac{1+\gamma_5}{2} + m_s\frac{1-\gamma_5}{2}]bF^{\mu\nu} \qquad (47)$$

In order to obtain the transition amplitude, we need to calculate the following matrix element:

$$< D_s^* \mid \overline{s}\sigma_{\mu\nu}(1 \pm \gamma_5)q^\nu b \mid B_c > \qquad (48)$$

At $q^2 = 0$, we can write this matrix element in terms of the two gauge invariant form factors $T_1(0)$ and $T_2(0)$

$$\begin{aligned}
< D_s^*(p,\varepsilon^{(D_s^*)}) \mid \overline{s}\sigma_{\mu\nu}q^\nu b \mid B_c(Q) > \; &= \; i\varepsilon_{\mu\alpha\beta\lambda}\varepsilon^{(D_s^*)\alpha}p^\beta Q^\lambda T_1(0), \\
< D_s^*(p,\varepsilon^{(D_s^*)}) \mid \overline{s}\sigma_{\mu\nu}q^\nu\gamma_5 b \mid B_c(Q) > \; &= \; [(m_{B_c}^2 - m_{D_s^*}^2)\varepsilon_\mu^{(D_s^*)} \\
&\quad - \; (\varepsilon^{(D_s^*)}.q)(p+Q)_\mu]T_2(0) \quad (49)
\end{aligned}$$

Using the relation

$$\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta} \qquad (50)$$

one can immediately obtain that $T_2(0)=\frac{1}{2}T_1(0)$. Then, we need to calculate only the form factor $T_1(0)$. For this aim, we define the following three point correlation function:

$$\Pi_{\mu\alpha} = -\int d^4x d^4y\, e^{i(Qx-py)} < 0 \mid T\{\overline{c}(y)\gamma_\alpha s(y)\overline{s}(0)\sigma_{\mu\nu}q^\nu b(0)\overline{c}(x)i\gamma_5 b(x)\} \qquad (51)$$

where $\overline{c}\gamma_\alpha s$ and $\overline{c}i\gamma_5 b$ are the interpolating currents of D_s^* and B_c mesons, respectively.

After inserting the hadrons full set with quantum numbers of corresponding interpolating currents (see also [12]), we obtain the following expression

for the phenomenological part of the correlation function:

$$\Pi_{\mu\alpha} = i\frac{f_{B_c}m_{B_c}^2}{(m_b + m_c)}\frac{f_{D_s^*}m_{D_s^*}}{(p^2 - m_{D_s^*}^2)(Q^2 - m_{B_c}^2)}\varepsilon_{\mu\alpha\beta\lambda}p^\beta Q^\lambda T_1(0) + \text{excited states.}$$

$$(52)$$

For the calculation of the QCD part, we write the Lorentz structure in the above correlator as:

$$\Pi_{\mu\alpha} = i\varepsilon_{\mu\alpha\beta\lambda}p^\beta Q^\lambda \Pi(p^2, Q^2) \tag{53}$$

where

$$\Pi^{per}(p^2, Q^2) = -\frac{1}{(2\pi)^2}\int d\tilde{s}ds'\frac{\rho^{per}(p^2, Q^2)}{(\tilde{s} - Q^2)(s' - p^2)} + \text{ subtraction terms} \tag{54}$$

The standard calculations lead to the following result for the pertubative part (bare-loop diagram):

$$\rho^{per}(s', \tilde{s}) = 4N_C[m_b m_c(A_1 + A_2 + I_0) - m_b^2 A_1 - 2A_3] \tag{55}$$

where

$$\begin{aligned}
A_1 &= \frac{2I_0}{(\tilde{s} - s')^2}[s'(\tilde{s} + m_c^2 - m_b^2) - \frac{1}{2}(\tilde{s} + s')(s' + m_c^2)], \\
A_2 &= \frac{2I_0}{(\tilde{s} - s')^2}[\frac{1}{2}(\tilde{s} + s')(m_b^2 - m_c^2 - \tilde{s}) + \tilde{s}(s' + m_c^2)], \\
A_3 &= I_0\frac{m_b^2[m_b^2 s' + (m_c^2 - s')(\tilde{s} - s')]}{2(\tilde{s} - s')^2}, \\
I_0 &= -\frac{1}{4(\tilde{s} - s')}
\end{aligned} \tag{56}$$

The integration regions over $\tilde{s}$ and s' are obtained from the following inequalities:

$$\begin{aligned}
m_c^2 &\leq s' \leq s_0', \\
s' - \frac{s'm_b^2}{m_c^2 - s'} &\leq \tilde{s} \leq \tilde{s}_0
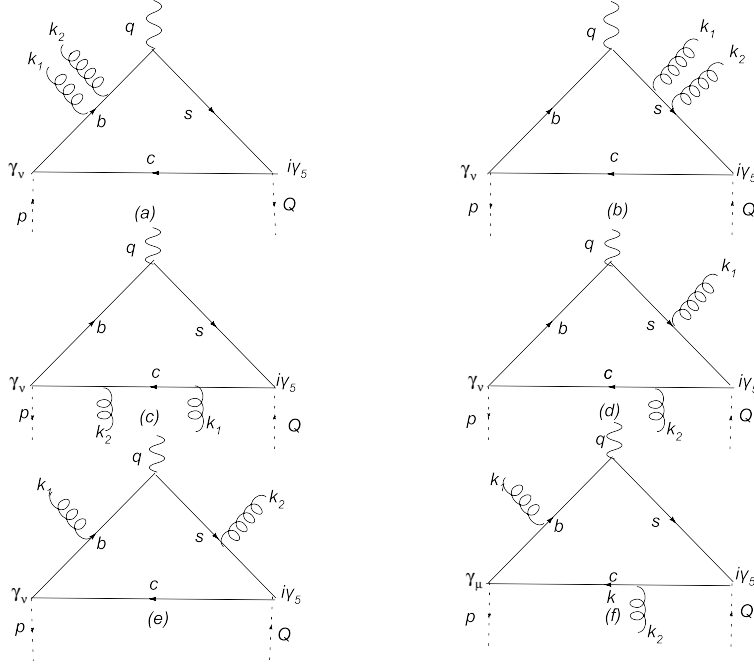\end{aligned} \tag{57}$$

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Figure 3: Feynmen diagrams for gluon corrections

The quark condensate terms give zero contribution after applying the double Borel transformation, with respect to the p^2 ($p^2 \rightarrow M_2^2$) and Q^2 ($Q^2 \rightarrow M_1^2$). Only the gluon condensates can contribute to the form factor. Fig. 3 shows such type of diagrams. After lengthy calculations for the gluon condensates contribution and equating the phenomenological and QCD parts and applying double Borel transformation with respect to the p^2 and Q^2, we find the following expression for the form factor $T_1(0)$:

$$
\begin{aligned}
T_1(0) &= -\frac{1}{(2\pi)^2}\frac{(m_b+m_c)}{f_{B_c}m_{B_c}^2 f_{D_s^*}m_{D_s^*}}[\int d\tilde{s}ds'\rho^{per}(s',\tilde{s})e^{-\frac{\tilde{s}}{M_1^2}}e^{-\frac{s'}{M_2^2}} \\
&+ M_1^2 M_2^2 <\frac{\alpha_s}{\pi}G^2> C_{G^2}]
\end{aligned}
\tag{58}
$$

where C_{G^2} is the Wilson coefficient of the gluon condensate and we thus have (see Fig. 3):

$$
C_{G^2} = C_{G^2}^a + C_{G^2}^b + C_{G^2}^c + C_{G^2}^d + C_{G^2}^e + C_{G^2}^f
\tag{59}
$$

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The explicit expressions for $C_{G^2}^i$ are given below as follows:

$$
\begin{aligned}
C_{G^2}^a &= 96 m_b \{ [m_c (I_0[1,3,1] + m_b^2 I_0[1,4,1] + I_1[1,3,1] + m_b^2 I_1[1,4,1] \\
&+ I_2[1,3,1] - m_b^2 I_2[1,4,1])] + m_b(I_1[1,3,1] + m_b^2 I_1[1,4,1] \\
&+ 2I_3[1,4,1]) \},
\end{aligned} \tag{60}
$$

$$
\begin{aligned}
C_{G^2}^b &= 16 \{ 2I_0[1,1,2] + 2m_b m_c I_0[1,1,3] + 2I_0[2,1,1] + 3m_b m_c I_0[2,1,2] \\
&+ 4m_c^2 I_0[2,1,2] + 4m_b m_c^3 I_0[2,1,3] + 2m_b m_c I_0[3,1,1] + 2m_c^2 I_0[3,1,1] \\
&+ 6m_b m_c^3 I_0[3,1,2] + 2m_c^4 I_0[3,1,2] + 2m_b m_c^5 I_0[3,1,3] + 2I_1[1,1,2] \\
&- 2m_b^2 I_1[1,1,3] + 2m_b m_c I_1[1,1,3] + 2I_1[2,1,1] - m_b^2 I_1[2,1,2] \\
&+ m_b m_c I_1[2,1,2] + 4m_c^2 I_1[2,1,2] - 4m_b^2 m_c^2 I_1[2,1,3] + 4m_b m_c^3 I_1[2,1,3] \\
&- 2m_b^2 I_1[3,1,1] + 2m_c^2 I_1[3,1,1] - 6m_b^2 m_c^2 I_1[3,1,2] + 4m_b m_c^3 I_1[3,1,2] \\
&+ 2m_c^4 I_1[3,1,2] - 2m_b^2 m_c^4 I_1[3,1,3] + 2m_b m_c^5 I_1[3,1,3] + 2m_b m_c I_2[1,1,3] \\
&+ I_2[2,1,1] + m_b m_c I_2[2,1,2] + 4m_b m_c^3 I_2[2,1,3] - 4m_c^2 I_2[3,1,1] \\
&+ 4m_b m_c^3 I_2[3,1,2] + 2m_b m_c^5 I_2[3,1,3] - 4I_3[1,1,3] - 4I_3[2,1,2] \\
&- 8m_c^2 I_3[2,1,3] - 8I_3[3,1,1] - 16m_c^2 I_3[3,1,2] - 4m_c^4 I_3[3,1,3] \} \\
&- 32M_2^2 \frac{d}{dM_2^2} \{ M_2^2 [I_0[2,1,2] + 2m_b m_c I_0[2,1,3] + 2m_b m_c I_0[3,1,2] \\
&+ m_c^2 I_0[3,1,2] + 2m_b m_c^3 I_0[3,1,3] + I_1[2,1,2] - 2m_b^2 I_1[2,1,3] \\
&+ 2m_b m_c I_1[2,1,3] - 2m_b^2 I_1[3,1,2] + m_b m_c I_1[3,1,2] + m_c^2 I_1[3,1,2] \\
&- 2m_b^2 m_c^2 I_1[3,1,3] + 2m_b m_c^3 I_1[3,1,3] - I_2[2,1,2] + 2m_b m_c I_2[2,1,3] \\
&- 2I_2[3,1,1] + m_b m_c I_2[3,1,2] - m_c^2 I_2[3,1,2] + 2m_b m_c^3 I_2[3,1,3] \\
&- 4I_3[2,1,3] - 6I_3[3,1,2] - 4m_c^2 I_3[3,1,3]] \} \\
&- 32M_2^4 (\frac{d^2}{dM_2^2})^2 \{ M_2^4 [m_b^2 I_1[3,1,3] + 2I_2[3,1,2] - m_b m_c (I_0[3,1,3] \\
&+ I_1[3,1,3] + I_2[3,1,3]) + 2I_3[3,1,3]] \} ,
\end{aligned} \tag{61}
$$

$$
C_{G^2}^c = 96 m_c \{ -(m_b^2 m_c I_1[4,1,1]) + m_b(I_0[3,1,1] + m_c^2 I_0[4,1,1]
$$

$$+ \quad I_1[3,1,1] + m_c^2 I_1[4,1,1] + I_2[3,1,1] + m_c^2 I_2[4,1,1]) - 2m_c I_3[4,1,1]\},$$

$$(62)$$

$$
\begin{aligned}
C_{G^2}^d \quad = \quad & -32 m_b m_c \{ I_0[2,1,2] + I_0[3,1,1] + m_c^2 I_0[3,1,2] + I_1[2,1,2] \\
+ \quad & m_c^2 I_1[3,1,2] + I_2[2,1,2] + m_c^2 I_2[3,1,2] - 4 I_3[3,1,2] \\
- \quad & M_2^2 \frac{d}{dM^2} [M_2^2 (I_0[3,1,2] + I_1[3,1,2] + I_2[3,1,2])] \} + 16 \{ m_b^2 I_1[2,1,2] \\
+ \quad & I_2[2,1,1] - m_b m_c (I_0[2,1,2] + I_1[2,1,2] + I_2[2,1,2]) + 4 I_3[2,1,2] \},
\end{aligned}
$$

$$(63)$$

$$
\begin{aligned}
C_{G^2}^e \quad = \quad & 16 \{ 2 I_0[1,1,2] + I_0[1,2,1] + 2 m_b^2 I_0[1,2,2] + m_b m_c I_0[1,2,2] \\
+ \quad & m_c^2 I_0[2,1,2] + m_b m_c I_0[2,2,1] + m_c^2 I_0[2,2,1] + m_b^2 m_c^2 I_0[2,2,2] \\
+ \quad & m_b m_c^3 I_0[2,2,2] + I_1[1,1,2] + I_1[1,2,1] + m_b^2 I_1[1,2,2] \\
+ \quad & m_b m_c I_1[1,2,2] - I_1[2,1,1] - 2 m_b m_c I_1[2,1,2] - m_b^2 I_1[2,2,1] \\
+ \quad & m_c^2 I_1[2,2,1] - 2 m_b^3 m_c I_1[2,2,2] + m_b m_c^3 I_1[2,2,2] + I_2[1,1,2] \\
+ \quad & + I_2[1,2,1] + m_b^2 I_2[1,2,2] + m_b m_c I_2[1,2,2] - 2 m_b m_c I_2[2,2,1] \\
+ \quad & m_c^2 I_2[2,2,1] + m_b m_c^3 I_2[2,2,2] - 2 I_3[2,1,2] - 4 I_3[2,2,1] \\
- \quad & 2 m_b^2 I_3[2,2,2] - 4 m_b m_c I_3[2,2,2] \} \\
- \quad & 16 M_1^2 \frac{d}{dM_1^2} \{ M_1^2 [I_2[2,2,1] + 2 I_3[2,2,2]] \} \\
+ \quad & 16 M_2^2 \frac{d}{dM_2^2} \{ M_2^2 [-m_b m_c I_0[2,2,2] + I_1[2,1,2] + m_b^2 I_1[2,2,2] \\
- \quad & m_b m_c I_1[2,2,2] + I_2[2,1,2] + I_2[2,2,1] + m_b^2 I_2[2,2,2] \\
- \quad & m_b m_c I_2[2,2,2] + 2 I_3[2,2,2]] \} + 16 \{ I_1[1,1,2] + m_b^2 I_1[1,2,2] \\
+ \quad & I_2[1,2,1] - m_b m_c (I_0[1,2,2] + I_1[1,2,2] + I_2[1,2,2]) \\
+ \quad & 2 I_3[1,2,2] \} \, ,
\end{aligned}
$$

$$(64)$$

$$C_{G^2}^f \quad = \quad 16 \{ 2 I_0[1,2,1] + 2 I_0[2,1,1] + 2 m_b^2 I_0[2,2,1]$$

$$
\begin{aligned}
+ \quad & 6m_b m_c I_0[2,2,1] + 2m_c^2 I_0[2,2,1] + 2I_1[1,2,1] \\
- \quad & 5I_1[2,1,1] - 5m_c^2 I_1[2,2,1] + 6m_b m_c I_1[2,2,1] + 2m_c^2 I_1[2,2,1] \\
+ \quad & 2I_2[1,2,1] - I_2[2,1,1] - m_b^2 I_2[2,2,1] + 6m_b m_c I_2[2,2,1] \\
+ \quad & 2m_c^2 I_2[2,2,1] - 14 I_3[2,2,1]\} - 32 M_1^2 \frac{d}{dM_1^2} \{ M_1^2 [I_0[2,2,1] \\
+ \quad & I_1[2,2,1] + I_2[2,2,1]]\}
\end{aligned}
\tag{65}
$$

and the for explicit form of the $I_i[a,b,c]$, we obtain:

$$
\begin{aligned}
I_0[a,b,c] &= \frac{(-1)^{a+b+c}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{2-a-c} \\
&\quad \times \mathcal{U}_0(a+b+c-4, 1-c-b) , \\
I_1[a,b,c] &= \frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{3-a-c} \\
&\quad \times \mathcal{U}_0(a+b+c-5, 1-c-b) , \\
I_2[a,b,c] &= \frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{2-a-c} \\
&\quad \times \mathcal{U}_0(a+b+c-5, 1-c-b) , \\
I_3[a,b,c] &= \frac{(-1)^{a+b+c+1}}{32\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{3-a-c} \\
&\quad \times \mathcal{U}_0(a+b+c-6, 2-c-b)
\end{aligned}
\tag{66}
$$

The function $\mathcal{U}_0(i,j)$, also, is given by:

$$
\mathcal{U}_0(i,j) = \int_0^\infty dy (y + M_1^2 + M_2^2)^i y^j \, exp\left[-\frac{B_{-1}}{y} - B_0 - B_1 y \right] ,
\tag{67}
$$

where

$$
\begin{aligned}
B_{-1} &= \frac{m_b^2}{M_1^2} \left[M_1^2 + M_2^2 \right] , \\
B_0 &= \frac{1}{M_1^2 M_2^2} \left[M_1^2 m_c^2 + M_2^2 (m_c^2 + m_b^2) \right] , \\
B_1 &= \frac{m_c^2}{M_1^2 M_2^2}
\end{aligned}
\tag{68}
$$

5 Numerical analysis

In this section, we present our numerical analysis for the form factors. From the sum rule expressions of these form factors, we see that the condensates, leptonic decay constants of B_c and D_s^* mesons, continuum thresholds s_0, $\tilde{s}_0$ and s_0', the relevant parameters in photon distribution amplitudes (DA's) and Borel parameters M_B^2, M_1^2 and M_2^2 are the main input parameters. In further numerical analysis, we choose the value of the condensates at a fixed renormalization scale of about 1 GeV. The values of the condensates are[22]: $< \overline{\psi}\psi \mid_{\mu=1\ GeV}>= -(240 \pm 10\ MeV)^3$, $< \overline{s}s >= (0.8 \pm 0.2) < \overline{\psi}\psi >$ and $m_0^2 = 0.8\ GeV^2$. The quark and mesons masses are taken to be $m_c(\mu = m_c) = 1.275 \pm 0.015\ GeV$, $m_s(1\ GeV) \simeq 142\ MeV$ [23] , $m_b = (4.7 \pm 0.1)\ GeV$ [22] , $m_{D_s^*} = 2.112\ GeV$ and $m_{B_C} = 6.258\ GeV$. For the values of the leptonic decay constants of B_C and D_s^* mesons, we use the results obtained from the two-point QCD analysis: $f_{B_C} = 0.35 GeV$ [26, 27, 28] and $f_{D_s^*} = 266 \pm 32\ MeV$[24]. The relevant parameters in photon distribution amplitudes (DA's) are taken to be $\chi = 3.15 \pm 0.3 GeV^{-2}$, $\kappa = 0.2$, $\kappa^+ = 0$, $\zeta_1 = 0.4$, $\zeta_1^+ = 0$, $\zeta_2 = 0.3$, $\zeta_2^+ = 0$, $f_{3\gamma} = -(4 \pm 2) \times 10^{-3} GeV^2$, $w_\gamma^A = -2.1 \pm 1.0$, $w_\gamma^V = 3.8 \pm 1.8$ [20, 21, 25]. The threshold parameters are also determined from the two-point QCD sum rules: $s_0 = 8\ GeV^2$, $\tilde{s}_0 = 45\ GeV^2$, $s_0' = 8\ GeV^2$ [2, 24, 29]. The Borel parameters M_B^2, M_1^2 and M_2^2 are auxiliary quantities and, therefore the results of physical quantities should not depend on them. In the QCD sum rule method, OPE is truncated at finite order, leaving a residual dependence on the Borel parameters. For this reason, the working regions for the Borel parameters should be chosen such that in these regions the form factors are practically independent of them. The working regions for the Borel parameters M_B^2, M_1^2 and M_2^2 can be determined on the condition that, on the one side, the continuum contribution should be

small, and on the other side, the contribution of the operator with the highest dimension should be small. As a result of the above-mentioned requirements, the working regions for this transition are obtained to be:

$$4\ GeV^2 < M_B^2 < 10\ \texttt{GeV}^2,\ 10\texttt{GeV}^2 \leq M_1^2 \leq 25\texttt{GeV}^2,$$

$$4\texttt{GeV}^2 \leq M_2^2 \leq 10\texttt{GeV}^2. \tag{69}$$

Now, by calculating the total decay widths and taking $\mid V_{cs} \mid = 0.957 \pm 0.017$, $\mid V_{cb} \mid = 0.0416 \pm 0.0006$, $\mid V_{tb} \mid = 0.77^{+0.18}_{-0.24}$, $\mid V_{ts} \mid = (40.6 \pm 2.7) \times 10^{-3}$ [30], $< \frac{\alpha_s}{\pi} G^2 > = 0.012 GeV^4$ [29], $C_7(\mu = m_c) = -0.0068 - 0.02i$ [13] and $\tau_{B_c} = 0.52 \times 10^{-12} s$ [31], we obtain the numerical results of the electromagnetic penguin(EP), weak annihilation(WA) and total branching ratios for this decay as follows:

$$\mathbf{B}^{(EP)}(B_c \rightarrow D_s^* \gamma) = 3.468 \times 10^{-6}$$

$$\mathbf{B}^{(WA)}(B_c \rightarrow D_s^* \gamma) = 1.557 \times 10^{-5}$$

$$\mathbf{B}^{(Total)}(B_c \rightarrow D_s^* \gamma) = 2.462 \times 10^{-5} \tag{70}$$

From the above results, we see that the weak annihilation contribution to the total branching ratio is about 4.48 times greater than that of the electromagnetic penguin diagram. Here, it is observed that the difference between the total branching ratio with sum of the weak annihilation and electromagnetic penguin branching ratios comes from the cross term in total decay width. Also our result for the total branching ratio shows that the $B_c \rightarrow D_s^* \gamma$ decay can be measured at LHC.

Now, we compare our results of the $B_c \rightarrow D_s^* \gamma$ to the results of the perturbative QCD [15], relativistic independent quark model [16], pertubative QCD in standard model (SM (PQCD)) [17] , multi scale walking techni-

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color (MWTCM) [17] and topcolor assisted MWTCM (TAMWTCM) [17] for $\tau_{B_c} = 0.52 \times 10^{-12}s$ as shown in Table (1).

	$\mathbf{B}^{EP}(B_c \to D_s^*\gamma)$	$\mathbf{B}^{WA}(B_c \to D_s^*\gamma)$	$\mathbf{B}^{Total}(B_c \to D_s^*\gamma)$
Present study	3.468×10^{-6}	1.557×10^{-5}	2.462×10^{-5}
PQCD	3.70×10^{-6}	4.94×10^{-6}	1.14×10^{-5}
RIQM	2.40×10^{-5}	4.51×10^{-5}	1.39×10^{-4}
MWTCM	$(0.68 - 3.42)10^{-4}$	$(0.74 - 0.81)10^{-6}$	$(0.74 - 3.57) \times 10^{-4}$
TAMWTCM	$(5.18 - 7.23)10^{-7}$	$(7.24 - 8.13)10^{-7}$	$(1.78 - 9.95) \times 10^{-6}$
SM(PQCD)	1.73×10^{-7}	5.89×10^{-7}	7.83×10^{-7}

Table 1: Comparison of the branching ratio for $B_c \to D_s^*\gamma$ decay based on the $\tau_{B_c} = 0.52 \times 10^{-12}s$.

Looking at this table, it is seen that there is a good agreement between the present study and the PQCD [15], in order of magnitude for the total branching ratio. However, our result is approximately one order of magnitude less than that of the RIQM and MWTCM. Also, it is one order of magnitude and two orders of magnitude greater than that of the TAMWTCM and SM(PQCD) [17], respectively. The ratio of $\mathbf{B}^{WA}/\mathbf{B}^{EP}$ for the present work, PQCD [15], RIQM, SM(PQCD) [17], TAMWTCM, MWTCM are 4.48, 1.34, 1.9, 3.4, 1.23 and 0.01, respectively. As a result of the above discussions, we can say that in the QCD sum rules (present study), relativistic independent quark model, perturbative QCD and TAMWTCM approaches, the weak annihilation contribution to the total branching ratio dominates the contribution coming from the electromagnetic penguin diagram, but this is not true only for the MWTCM approach. The presence of the pseudo Goldstone bosons in the MWTCM leads to a discrepancy between this model and the other two models in [17] (for more details see[17]) and a part of inconsistency in the results of the different methods may be related to the different

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magnitudes of the input parameters, getting from different references; e.g. we use $m_c(\mu = m_c) = 1.275 \pm 0.015 \ GeV$ for the c quark masses while the authors of [17] use $m_c = 1.6 \ GeV$ and also to the nature of the methods and their accuracy.

In this step, for the analysis of $B_c \rightarrow D^*\gamma$, in the entire calculations we replace the s quark with the d quark. Making $m_{D_s^*} \rightarrow m_{D^*}$, $f_{D_s^*} \rightarrow f_{D^*}$, $V_{ts} \rightarrow V_{td}$, $V_{cs} \rightarrow V_{cd}$ changes and taking $m_{D^*} = 2.010 \ GeV$, $f_{D^*} = 0.23 \pm 0.02 \ GeV$ [32], $V_{cd} = 0.230 \pm 0.011$, $V_{td} = (7.4 \pm 0.8) \times 10^{-3}$ [30] and $m_d = 5 \ MeV$ we obtain the numerical results as below:

$$\mathbf{B}^{(EP)}(B_c \rightarrow D^*\gamma) = 1.151 \times 10^{-7}$$
$$\mathbf{B}^{(WA)}(B_c \rightarrow D^*\gamma) = 2.162 \times 10^{-6}$$
$$\mathbf{B}^{(Total)}(B_c \rightarrow D^*\gamma) = 2.786 \times 10^{-6} \tag{71}$$

These results also enhance the importance of the weak annihilation contribution to the total branching ratio in comparing with the electromagnetic penguin diagram ones for the $B_c \rightarrow D^*\gamma$. Finally, we compare our results to the relativistic independent quark model (RIQM) [16] for $\tau_{B_c} = 0.52 \times 10^{-12}s$ in Table (2).

From the Table 2, it is also seen a good agreement in the order of magnitude

	$\mathbf{B}^{EP}(B_c \rightarrow D^*\gamma)$	$\mathbf{B}^{WA}(B_c \rightarrow D^*\gamma)$	$\mathbf{B}^{Total}(B_c \rightarrow D^*\gamma)$
Present study	1.151×10^{-7}	2.162×10^{-6}	2.786×10^{-6}
RIQM [16]	5.70×10^{-7}	1.33×10^{-6}	3.64×10^{-6}

Table 2: Comparison of the branching ratio for $B_c \rightarrow D^*\gamma$ decay based on the $\tau_{B_c} = 0.52 \times 10^{-12}s$.

between the present study and the relativistic independent quark model.

In conclusion, the present study concentrated on the radiative $B_c \to D_s^* \gamma$ and $B_c \to D^* \gamma$ decays in the framework of QCD sum rules. The form factors responsible for these decays were calculated. The branching ratio for this decays were estimated. The results show that the $B_c \to D_s^* \gamma$ case can be measured at LHC in the near future.

6 Acknowledgment

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